

Potential between adjoint sources in arbitrary representations

Abstract

The potential between sources in arbitrary representations of the gauge group is studied on an anisotropic lattice in a spherical model approximation. It is shown analytically that for half-integer j and j' in the confinement phase the potential rises linearly, whereas for integer j and half-integer j' it rises infinitely which means a strong suppression of the combination of such states. For integer j and j' the potential shows Debye screening and Coulomb behavior in the deconfinement phase. It is also shown, that $\langle \chi^{(j)} \rangle \sim \langle \chi \rangle^{2j}$ when $\langle \chi \rangle \gtrsim 1$ and is in agreement with the mean field theory prediction, and $\langle \chi^{(j)} \rangle \sim \langle \chi \rangle$ for $\langle \chi \rangle \lesssim 1$ which agrees with MC experiment. String tension model-computed for sources invariant under center group transformations demonstrates Casimir scaling in the intermediate distance regime and turns into zero at large distances.

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1 Introduction.

Statistical QCD evaluated on the lattice by means of computer simulation is perhaps the only case in statistical physics where the critical behavior can be calculated from the first principles of dynamics [1, 2, 3]. The power of the approach is best seen in the thermodynamics of pure $SU(N)$ gauge theory, i.e., for the systems consisting of gluons only [1, 4, 5]. The main features considered are the deconfinement transition and the properties of the hot gluon plasma [6]. Lattice gauge theory (LGT) provides a powerful tool to compute nonperturbative properties of strong interactions, such as a potential between pairs of infinitely heavy sources.

It is known from Monte Carlo simulations that probes in representations insensitive to the centre gauge group transformation $Z(N)$ yield a screened potential, while those sensitive to this transformations provide a linearly rising one $F \approx \alpha R$ (where α - string tension, R - distance between sources) [7, 8, 9]. As indicated in [10], the potential between sources transforming as arbitrary irreducible representations of the gauge group provides a good laboratory for testing the dynamics behind screening and confining mechanisms. In particular, one can measure the relevant distance scales, i.e. those that separate “short distances” (essentially perturbative physics), from the “long distances”. The dynamics of the intermediate region has been found quite rich in many aspects. Indeed, some very simple numerical simulations have shown that adjoint sources, for example, may feel a linearly rising potential at intermediate distances [11, 12, 13, 14, 15]. Such sources appear to “deconfine” at precisely the same critical temperature T_c at which fundamental sources deconfine [16, 17]. The string tension is representation-dependent and appears to be roughly proportional to the eigenvalue of the quadratic Casimir operator [11, 12, 18, 13].

As the distance between color sources with zero “n-ality” increases they are eventually screened by gluons. This asymptotic regime extends from the color-screening length to infinity, and in the case of $SU(2)$ gauge group the string tensions become equal for half-integer j -s and turn into zero for integer ones. In particular, the string between quarks in an adjoint representation must

break at some distance that presumably depends on the mass of "gluelumps" (i.e. on the energy of a gluon bound to a massive adjoint quark) [19].

From universality hypothesis it follows, in particular, that all higher representations sensitive to the center of the gauge group should be *equivalent* order parameters with the same critical behavior as the fundamental representation. However different behavior was found for the higher representations at $SU(2)$ lattice gauge theory in $(3+1)$ dimensions [16, 17].

The purpose of this work is an analytical investigation of the potential between sources in arbitrary representations of the gauge group near the critical point on an anisotropic lattice in a spherical model approximation.

The application of spherical model in statistical physics has a long history since the time it had been introduced to investigate critical phenomena in the ferromagnet [20] and until recently (see, e.g., [21, 22]). Stanley [23] established the correspondence between the spherical and the Heisenberg models.

Although this model is of no direct experimental relevance, it may provide useful insight since many physical quantities of interest can be exactly evaluated with its help. In this context, the spherical model is quite a useful tool in providing explicit checks of general concepts in critical phenomena, see [24, 25, 26, 27]. Recently [21] it was successfully used for studying the transitions between a paramagnetic, a ferromagnetic and an ordered incommensurate phase (Lifshitz point). Models of this kind were investigated extensively (see recent review in [28]). The spherical model predicts reasonable values for critical exponents. Moreover, a 'basic' set of exponent relations is also satisfied by spherical model for $2 < d < 5$ [29].

2 LGT on anisotropic lattice.

The simplest way to increase the accuracy in numerical study of LGT at large distances and high temperatures is to use anisotropic lattices in which the temporal spacing is much smaller than that

in the spatial directions ($\xi = a_\sigma/a_\tau \gg 1$, the Hamiltonian limit). This approach becomes a very popular technique at the present time since it allows Monte Carlo calculations to be carried out with reasonable computational resources and reap many of the benefits of fine lattices, while still using cheap coarse spatial ones. There are convincing arguments [30, 31] that in the anisotropic Wilson gauge action no coefficients are to be tuned in order to restore space-time exchange symmetry up to $O(a^2)$ errors. On such lattices heavy quarks will not suffer large lattice artifacts as long as their masses are small in the units of a_τ .

To compute the partition function on a lattice of a size $N_\tau \times N_\sigma^3$ (N_τ is the temporal extent, N_σ^3 is the spatial extent of a lattice) after [32] (see also [33, 34]) we use the anisotropic lattice

$$S_G = \beta_\tau S_E + \beta_\sigma S_M, \quad (1)$$

where S_E is electric and S_M magnetic part of the gluodynamics action and

$$\beta_\tau \equiv \beta \tilde{\xi}(\xi, \beta) \gg \beta_\sigma \equiv \beta / \tilde{\xi}(\xi, \beta), \quad \beta \equiv \frac{2N}{g^2}, \quad (2)$$

with

$$\xi = a_\sigma/a_\tau. \quad (3)$$

In the weak coupling region $\beta_{\tau,\sigma}/\beta \approx 1 + O(1/\beta)$ [35, 36] and in this case $\tilde{\xi}$ does not differ essentially from ξ . Moreover, in the “naive” limit

$$U_\nu(x) \simeq 1 + i a_\nu g_\nu A_\nu^c(x) T_c \quad (4)$$

and one must put exactly $\tilde{\xi} = \xi$. However, in LGT the effective values of the integration variables $U_\nu(x)$ are far enough from the “naive” limit (especially in the deconfinement region). The deviation of $\tilde{\xi}$ from ξ is regarded as ‘quantum corrections’ or ‘renormalisation’. The behavior of $\tilde{\xi}(\xi, \beta)$ in the vicinity $\xi \sim 1$ has been studied both perturbatively [37] and by non-perturbative methods [38, 39, 40, 41, 42, 43]. Since we cannot exclude, that in the area of the asymptotically large ξ the

behavior of $\tilde{\xi}$ may be essentially different, we will not specify the dependence of $\tilde{\xi}$ on ξ , but only assume that $\tilde{\xi}(\xi, \beta)$ may be made arbitrary large by increasing ξ at fixed β .

It is well known [44, 45] (see a review in [46]) that there is a variety of ways to get (in accordance with a universality hypothesis [32]) the effective action

$$-S_{eff} = \gamma(\beta, \xi, N_\tau) \text{Re} \sum_{l=1}^3 \sum_x^{N_\sigma^3} \chi_x \chi_{x+l}^*, \quad (5)$$

expressed in terms of the Polyakov lines $\chi_x^{(j)} = \text{Tr} \prod_{t=1}^{N_\tau} U_0^{(j)}(x, t)$ in j -representation of $SU(N)$ group (we omit index j for fundamental representation). The crucial assumption for obtaining (5) is to discard the magnetic part ($S_M \sim 1/\tilde{\xi}$) of action. Then $S_G(U) \simeq S_E$ with

$$S_E = -\beta \tilde{\xi} \sum_x \sum_{\nu=1}^3 \text{Re} \text{Tr} \left\{ U_0(x) U_\nu(x+0) U_0^\dagger(x+\nu) U_\nu^\dagger(x) - 1 \right\}. \quad (6)$$

As a rule it can be done in a strong coupling approximation [44, 45]. However, as it is pointed out in [32] at high 'temperatures' $T_{SY} = \xi/(a_\tau N_\tau) \equiv \xi T$ and for the couplings one gets $\beta_\sigma \sim 1/T_{SY}$ and $\beta_\tau \sim T_{SY}$. Hence we may assume that for finite T the magnetic part of action may be neglected [34] compared to the electric one ($S_E \sim \xi$) in a Hamiltonian limit ($\xi \gg 1$) even at low temperatures T . Such an assumption doesn't look as harmless as in a strong coupling case. Indeed, there are serious reasons to believe that the magnetic part of the action may be of crucial importance for creating the confining forces [47, 48, 49], even at high temperatures [50]. Therefore, strictly speaking, QCD without a magnetic part may be considered in a weak coupling area only as a specific ('toy') model.

Summarizing for the partition function with static sources η_x we may eventually write

$$Z \sim \int \exp \left\{ \text{Re} \left(\gamma \sum_{x,l} \chi_x \chi_{x+l}^* + \sum_x \eta_x \cdot \chi_x^* \right) \right\} \prod_x^{N_\sigma^3} d\mu_x. \quad (7)$$

3 $SU(2)$ gauge model

In a simple case of $SU(2)$ gauge group one may write

$$d\mu_x = \sin^2\left(\frac{\varphi_x}{2}\right) \frac{d\varphi_x}{2\pi} \Big|_{-2\pi}^{2\pi}; \quad \chi_x = \chi_x^* = 2 \cos \frac{\varphi_x}{2}, \quad (8)$$

so by introducing new variables: $\sigma_x = 2 \cos \frac{\varphi_x}{2}$ and $\tilde{\sigma}_x = 2 \sin \frac{\varphi_x}{2}$ the partition function (7) can be rewritten as

$$Z \sim \int_{-\infty}^{+\infty} \exp \left\{ \gamma \sum_{x,l} \sigma_x \sigma_{x+l} + \sum_x \eta_x \sigma_x \right\} \prod_x \delta(\sigma_x^2 + \tilde{\sigma}_x^2 - 4) d\sigma_x \tilde{\sigma}_x^2 d\tilde{\sigma}_x. \quad (9)$$

To compute the partition function we use the spherical model approximation¹:

$$\begin{aligned} \prod_x \delta(4 - \sigma_x^2 + \tilde{\sigma}_x^2) &\rightarrow \delta\left(4N - \sum_x (\sigma_x^2 + \tilde{\sigma}_x^2)\right) \\ &= \int_{c-i\infty}^{c+i\infty} \frac{ds}{2\pi i} e^{s(4N - \sum_x (\sigma_x^2 + \tilde{\sigma}_x^2))}, \end{aligned} \quad (10)$$

where constant c is chosen so that the integration path is placed at the right side of all singularities of the integrand to ensure the legitimacy of interchanging the integration order over ds and $d\sigma_x d\tilde{\sigma}_x$.

Therefore we can rewrite (9) as

$$Z = \text{const}' \int ds e^{4sN} \int_{-\infty}^{+\infty} \exp(-\sigma_x A_{xx'} \sigma_{x'} + \eta_x \sigma_x) d\sigma_x \int_{-\infty}^{+\infty} e^{-s\tilde{\sigma}_x^2} \tilde{\sigma}_x^2 d\tilde{\sigma}_x, \quad (11)$$

where

$$A_{xx'} = s\delta_{x,x'} - \gamma \sum_{l=1}^d \delta_{x+l,x'}. \quad (12)$$

It is clear that integration over $\tilde{\sigma}_x$ can be done trivially

$$\int \prod_x^{N_\sigma^3} e^{-s\tilde{\sigma}_x^2} \tilde{\sigma}_x^2 d\tilde{\sigma}_x = \exp \left\{ -N_\sigma^3 \frac{3}{2} \ln(s - 3\gamma) + \text{const} \right\}. \quad (13)$$

Integration over σ_x may be fulfilled after Fourier transformation²

$$\sigma_x = \sum_k \zeta_k e^{iqx}, \quad q_l \equiv \frac{2\pi k_l}{N_\sigma}; \quad l = 1; 2; 3; \quad k_l = 0, \dots, N_\sigma - 1 \quad (14)$$

¹ We want to stress, that the spherical model approximation does not break the global $Z(2)$ - invariance of the model.

² Discrete variables q_l for $N_\sigma \gg 1$ can be considered as continuous: $0 \leq q_l < 2\pi$.

which diagonalizes bilinear form

$$\sum_{x,x'} \sigma_x A_{xx'} \sigma_{x'} = N_\sigma^3 \sum_k |\zeta_k|^2 A(q), \quad (15)$$

where

$$A(q) = s - \gamma \sum_{l=1}^d \cos q_l. \quad (16)$$

Now we can do the integration over ζ_k and write for partition function

$$Z = \text{const} \int_{c-i\infty}^{c+i\infty} ds \exp \left(\sum_{x,x'} \eta_x A_{xx'}^{-1} \eta_{x'} + N_\sigma^3 \left(4s - b(s) - \frac{3}{2} \ln s \right) \right), \quad (17)$$

where

$$b(s) = \frac{1}{2} \text{Sp} \{ \ln A \} = \frac{1}{2} \int_0^{2\pi} \left(\frac{dq}{2\pi} \right)^d \ln \left(s - \gamma \sum_{l=1}^d \cos q_l \right), \quad (18)$$

and

$$3\gamma A_{xy}^{-1} = \int_0^{2\pi} \frac{e^{-iq_3(x-y)} \left(\frac{dq}{2\pi} \right)^3}{\frac{s}{3\gamma} - \frac{1}{3} \sum_{l=1}^3 \cos q_l} = R_3 \left(\frac{s}{3\gamma}; x-y \right), \quad (19)$$

with

$$R_3 \left(\frac{s}{3\gamma}; x \right) = \int_0^\infty e^{-t \frac{s}{3\gamma}} I_x \left(\frac{t}{3} \right) I_0^2 \left(\frac{t}{3} \right) dt. \quad (20)$$

and $I_n(x)$ - modified Bessel function.

Taking into account that

$$\sum_x A_{xy}^{-1} = \sum_y A_{xy}^{-1} = \frac{1}{s - 3\gamma}, \quad (21)$$

we obtain for the uniform sources $\eta_x \equiv \eta_0$

$$Z \simeq \text{const} \int_{c-i\infty}^{c+i\infty} ds \exp \{ N \Phi(s) \}, \quad (22)$$

with

$$\Phi(s) \equiv 4s - b(s) - \frac{3}{2} \ln s + \frac{\eta_0^2}{4(s - \gamma d)}. \quad (23)$$

To calculate (22) we use the steepest descent method that in the limit $N_\sigma^3 \rightarrow \infty$ gives an accuracy of order $O\left(\frac{1}{N_\sigma^3}\right)$. The saddle point s_0 is determined by $\Phi'(s_0) = 0$ and considering that one may write [29] for $\frac{s_0}{3\gamma} - 1 \equiv \varepsilon \simeq 0$.

$$R_3(\varepsilon + 1; 0) \approx \frac{3}{2} - \frac{3}{\pi} \sqrt{\frac{3}{2}} \cdot \sqrt{\varepsilon}, \quad (24)$$

we get

$$\Phi' = 4 - \frac{1}{2} \left(\frac{1}{2\gamma} - \frac{\sqrt{\frac{3}{2}}}{\pi\gamma} \sqrt{\varepsilon} \right) - \frac{1}{2\gamma(1+\varepsilon)} - \frac{\eta_0^2}{4(3\gamma)^2 \varepsilon^2}. \quad (25)$$

Hence for $\gamma < \gamma_c$ the solution of (25) for $\eta = 0$ is given by

$$s_0 \simeq 3\gamma \left(1 + \frac{2}{3} (8\pi\gamma)^2 \left(\frac{\gamma_c}{\gamma} - 1 \right)^2 \theta(\gamma_c - \gamma) \right), \quad (26)$$

where the critical coupling γ_c is defined as the solution of $\Phi' = 0$ at the lowest value of $s_0 \rightarrow 3\gamma + 0$ (and $\eta \rightarrow 0$), which evidently gives $\gamma_c = 3/16$. In a vicinity of the critical point one may write for A_{00}^{-1}

$$\frac{1}{2} A_{0,0}^{-1} \simeq \frac{1}{4\gamma} - \frac{3}{2} \left(\frac{1}{2\gamma} - \frac{1}{2\gamma_c} \right) \theta(\gamma_c - \gamma). \quad (27)$$

It is easy to see, that in the deconfinement region ($\gamma > \gamma_c$) the solution of $\Phi' = 0$ with $\eta \equiv 0$ is absent. The only way to regain the solution in such region is to tend $s_0 \rightarrow 3\gamma$, because in this case the term $\frac{\eta_0^2}{(s_0 - 3\gamma)^2}$ must be preserved even in a limit $\eta_0 \rightarrow 0$. Now the solution

$$s_0 \simeq 3\gamma + \frac{\eta/2}{\sqrt{1 - \frac{\gamma_c}{\gamma}}} \rightarrow 3\gamma \quad (28)$$

sticks to the point 3γ so in this region we can write

$$\frac{1}{2} A_{00}^{-1} \simeq \frac{1}{4\gamma} \left(1 - \frac{\sqrt{\eta} \left(1 - \frac{\gamma_c}{\gamma} \right)^{-\frac{1}{4}}}{\sqrt{2\gamma\pi}} \right) \simeq \frac{1}{4\gamma}, \quad (29)$$

and

$$\langle \chi \rangle = \left[\left(\frac{1}{2} \eta_x A_{x,0}^{-1} \right) \right]_{s=s_0} \Big|_{\eta_0 \rightarrow 0} \simeq 2 \left(1 - \frac{\gamma_c}{\gamma} \right)^{\frac{1}{2}} \theta(\gamma - \gamma_c). \quad (30)$$

Hence in the deconfinement region $\langle \chi \rangle \neq 0$ but steadily approaches zero when $\gamma \rightarrow \gamma_c + 0$ in accordance with the well known fact, that $SU(2)$ - gluodynamics undergoes second order phase transition.

For the two point correlation function for $R \gg 1$ we get

$$\langle \chi_0 \chi_R \rangle = \frac{A_{0R}^{-1}(s_0)}{2} + \langle \chi \rangle^2 \simeq c \frac{e^{-\alpha R}}{R} + 4 \left(1 - \frac{\gamma_c}{\gamma} \right) \theta \left(1 - \frac{\gamma_c}{\gamma} \right), \quad (31)$$

where $c = \text{const}$ and string tension (in lattice units) is given by

$$\alpha \simeq 3\pi \left(1 - \frac{\gamma}{\gamma_c} \right) \theta \left(1 - \frac{\gamma}{\gamma_c} \right). \quad (32)$$

The expression for correlation function (31) as (20) is just the Fourier transform of the propagator $[D(q) + m^2]^{-1}$ where $D(q) \equiv \sum_{i=1}^d 4 \sin^2 \frac{q_i}{2}$ is the lattice laplacian in d -dimensional momentum space. In the perturbation theory expression (20) can be attributed to the lattice one-particle exchange. A basic non-perturbative feature of the high temperature plasma phase of QCD is the occurrence of the gluon screening mass [51], so parameter α , defined by (32) can be interpreted as screening mass as well. The expression (20) is not specific for the spherical model and appears in many popular approaches, for example in the matrix model (see e.g., [52]) and at the perturbation expansion ($g^2 \ll 1$) (see [53, 54]).

A more detailed case is considered in the Appendix. Here we present only the final results : We work in the parameter area $\gamma \sim \gamma_c$ and $R \gg 1$ where $A_{0,R}^{-1} / A_{0,0}^{-1} \ll 1$; and $\langle \chi \rangle \ll 1$ so we preserve only the terms with lower powers of $A_{0,R}^{-1} / A_{0,0}^{-1}$ and $\langle \chi \rangle$.

It is easy to see that in all cases the correlation function

$$C^{(j)(j')} \equiv \left\langle \chi_0^{(j)} \chi_R^{(j')} \right\rangle - \left\langle \chi^{(j)} \right\rangle \left\langle \chi^{(j')} \right\rangle \quad (33)$$

$j \text{ \& } j'$	$\left\langle \chi_0^{(j)} \chi_R^{(j')} \right\rangle$
both integer	$\left\langle \chi^{(j)} \right\rangle \left\langle \chi^{(j')} \right\rangle \exp \left\{ \left\langle \chi \right\rangle^2 A_{0,R}^{-1} c_1 + \left(j' j - \frac{1}{4} \right) \left(A_{0,R}^{-1} \right)^2 c_2 \right\}$
both half-int.	$\left\langle \chi^{(j)} \right\rangle \left\langle \chi^{(j')} \right\rangle + q(j'; j) \frac{2A_{0,R}^{-1}}{A_{0,0}}$
j int.; j' half-int.	$\left\langle \chi^{(j)} \right\rangle \left\langle \chi^{(j')} \right\rangle + \left\langle \chi^{(j-\frac{1}{2})} \right\rangle \left\langle \chi^{(j'-\frac{1}{2})} \right\rangle A_{0,R}^{-1} c_3$

Tab. I Pair correlation for states in higher representations

exponentially decreases in the confinement phase ($\langle \chi \rangle = 0$) and can be used for the computation of α , which plays a role of string tension when $\langle \chi^{(j)} \rangle \langle \chi^{(j')} \rangle = 0$ and may be interpreted as a screening mass for $\langle \chi^{(j)} \rangle \langle \chi^{(j')} \rangle \neq 0$.

4 Conclusions.

As it can be seen from TABLE 1, our model gives linearly rising free energy $F \sim \alpha R$ in the confinement phase for half-integer j and j' . The potential between states with j integer and j' half-integer (or vice versa) turns into infinity which means strong suppression of such combination of states. For integer j and j' we get the screening potential: $F \sim -\frac{e^{-2\alpha R}}{R}$. In the deconfinement phase ($\langle \chi \rangle \neq 0$) we get $F \sim -1/R$. We would like to stress here that in the deconfinement phase ($\langle \chi \rangle \neq 0$) for integer j and j' the screening mass is twice as much than in confinement ($\langle \chi \rangle = 0$) one.

As it was predicted by the mean field theory [55]

$$\left\langle \chi^{(j)} \right\rangle \sim \langle \chi \rangle^{2j}. \quad (34)$$

This result was confirmed in MC simulations [56] and as it can be seen from TABLE 1 roughly agrees with such data when $\langle \chi \rangle \gtrsim 1$. However, as it was pointed out in [57] in the vicinity of critical point the relation (34) does not agree with MC experiment. TABLE also shows, that our result (49)

$$\left\langle \chi^{(j)} \right\rangle \sim \langle \chi \rangle, \quad (35)$$

obtained in such area agrees with the data given in [57] and with theoretical predictions developed both in framework flux picture in a strong coupling approximation and within field theory (ϕ^4) on the basis of universality arguments [57].

Finally, there is the issue of the Casimir scaling (whose importance were emphasized in [11, 12]) of higher-representation string tensions in the intermediate distance regime. For values R far from asymptotic region ($\alpha R \lesssim 1$) we can write $A_{0,R}^{-1} \approx c_0 \cdot (1 - \alpha R)$ and $\langle \chi^{(j)} \chi^{(j')} \rangle \simeq \langle \chi^{(j)} \rangle \langle \chi^{(j')} \rangle \exp \left\{ -\alpha \cdot \left(j'j - \frac{1}{4} \right) cR + \text{const} \right\}$ where $c = 2c_2 c_0^2$ is independent either from R or from j and j' so it follows from (54) and (55):

$$\alpha(j; j') \sim \alpha \cdot \left(j'j - \frac{1}{4} \right). \quad (36)$$

Therefore, the adjoint sources may 'feel' a linearly rising potential at intermediate distances, and appear to "deconfine" at precisely the same critical point at which the fundamental sources really deconfine. This qualitatively agrees with Casimir scaling observed in such parameter region [11, 12]. It is easy to see, that at very large distance scales Casimir scaling breaks down and color screening is set.

As it was pointed long ago, sources trivially transformed under centre gauge group transformations can not be used as an order parameter. An average value of a pair of such sources also is not a good laboratory for critical phenomena studies, because the magnetization $M^{(j,j')} = \lim_{R \rightarrow \infty} \langle \chi_0^{(j)} \chi_R^{(j')} \rangle = \langle \chi^{(j)} \rangle \langle \chi^{(j')} \rangle$ differs from zero and masks the Debye term. Therefore $C_R^{j,j'} = \langle \chi_0^{(j)} \chi_R^{(j')} \rangle - \langle \chi^{(j)} \rangle \langle \chi^{(j')} \rangle \rightarrow 0$ with $R \rightarrow \infty$ so $\chi_0^{(j)}$ and $\chi_R^{(j')}$ are statistically independent at asymptotically large distances and long range order is absent either for $\langle \chi^{(j)} \rangle \langle \chi^{(j')} \rangle = 0$ or for $\langle \chi^{(j)} \rangle \langle \chi^{(j')} \rangle \neq 0$. On the other hand, the fact that the correlation function $C_R^{j,j'}$ exponentially decreases for all j and j' can be used for determination of string tension even in the cases when $\langle \chi^{(j)} \rangle$ or $\langle \chi^{(j')} \rangle$ are invariant under $Z(N)$ transformations.

We also wish to note that computing the Binder cumulant B , defined as [58]:

$$B = \frac{1}{2} \left(3 - \frac{\langle \chi^4 \rangle}{\langle \chi^2 \rangle^2} \right), \quad (37)$$

we get for a case $SU(2)$ gauge group

$$B = \left(1 + \frac{2A_{00}^{-1}}{\langle\chi\rangle^2}\right)^{-2}. \quad (38)$$

Therefore, in a vicinity of a critical point we get $B \approx 0$ because $\langle\chi\rangle = 0$ in the confinement region and in deconfinement region, if we are close enough to the critical point. Vanishing of B means that the distribution of χ is Gaussian, therefore the spherical model must give reliable results in both regions.

5 Appendix

The n - order correlations $\langle \prod_{k=1}^n \chi_{x_k}^{(j_k)} \rangle$ may be easily computed after Fourier transformation of $\chi^{(j)} = U_{2j} \left(\frac{\chi}{2} \right)$

$$U_r \left(\frac{\sigma}{2} \right) = \int_{-\infty}^{\infty} u_r(z) e^{i\sigma z} \frac{dz}{2\pi}; \quad u_r(z) = \int_{-\infty}^{\infty} U_r \left(\frac{\sigma}{2} \right) e^{-i\sigma z} d\sigma \quad (39)$$

and we get in a spherical model approximation

$$\begin{aligned} \left\langle \prod_{k=1}^n \chi_{x_k}^{(j_k)} \right\rangle &= \int_{-\infty}^{\infty} \left\langle \exp \left\{ \sum_x \eta_x \sigma_x \right\} \right\rangle \prod_{k=1}^n u_{2j_k}(z_k) \prod_{k=1}^n \frac{dz_k}{2\pi} \\ &= \int_{c-i\infty}^{c+i\infty} ds \int_{-\infty}^{\infty} \exp \{ -\sigma_x A_{x,x'} \sigma_{x'} + \eta_x \sigma_x \} \prod_{k=1}^n u_{2j_k}(z_k) \frac{dz_k}{2\pi} \prod_x d\sigma_x \end{aligned} \quad (40)$$

with

$$\eta_x = \eta_0 + i \sum_{k=1}^n z_k \delta_x^{x_k}; \quad \eta_0 \rightarrow 0. \quad (41)$$

Therefore after integration over σ_x

$$\left\langle \prod_{k=1}^n \chi^{(j_k)}(\varphi(x_k)) \right\rangle = \frac{\pi^{\frac{N^3}{2}}}{\sqrt{\det A}} \int_{-\infty}^{\infty} \prod_{k=1}^n u_{2j_k}(z_k) \exp \left\{ \frac{1}{4} \eta_x A_{x,x'}^{-1} \eta_{x'} \right\} \prod_{k=1}^n \frac{dz_k}{2\pi} \quad (42)$$

and taking into account (30)

$$\begin{aligned}
\lim_{\eta_0 \rightarrow 0} \frac{1}{4} \eta_x A_{x,x'}^{-1} \eta_{x'} &= -\frac{1}{4} z_k A_{x_k, x_n}^{-1} z_n + i z_k \left[\left(\frac{1}{2} \eta_x A_{x, x_k}^{-1} \right)_{s=s_0} \right]_{\eta_0 \rightarrow 0} \\
&= -\frac{1}{4} z_k \alpha_{k,r}^{-1} z_r + i \langle \chi \rangle \sum_k z_k,
\end{aligned}$$

we obtain inverting Fourier transformation

$$\left\langle \prod_{k=1}^n \chi_{x_k}^{(j_k)} \right\rangle = \overline{\prod_{k=1}^n U_{2j_k} \left(\frac{\sigma_k}{2} \right)}, \quad (43)$$

with

$$\overline{Q} \equiv \frac{\sqrt{\det \alpha}}{\pi^{\frac{n}{2}}} \int_{-\infty}^{\infty} Q \exp \left\{ - \sum_{k,r=1}^n (\sigma_k - \langle \chi \rangle) \alpha_{k,r} (\sigma_r - \langle \chi \rangle) \right\} \prod_{k=1}^n d\sigma_k, \quad (44)$$

where $\alpha_{k,r}$ is $n \times n$ matrix is defined by the condition $\alpha_{k,r}^{-1} = A_{x_k, x_r}^{-1}(s_0)$. In particular one can write

$$\begin{aligned}
\langle \chi^n \rangle &= (-D/2)^{\frac{n}{2}} H_n \left(\langle \chi \rangle / \sqrt{-2D} \right) \approx \\
&\begin{cases} \left(1 + k \langle \chi \rangle^2 / D \right) (D/2)^{\frac{n}{2}} n! / \left(\frac{n}{2} \right)! & \left[\frac{n}{2} \right] = \frac{n}{2}; \\ \langle \chi \rangle (D/2)^{\frac{n-1}{2}} n! / \left(\frac{n-1}{2} \right)! & \left[\frac{n}{2} \right] = \frac{n-1}{2}; \\ \langle \chi \rangle^n + \langle \chi \rangle^{n-2} D n (n-1) / 4 & \gamma > 2\gamma_c, \end{cases} \quad \gamma < 2\gamma_c, \quad (45)
\end{aligned}$$

where the dispersion $D \equiv \langle (\chi - \langle \chi \rangle)^2 \rangle$ is given by

$$D = \langle \chi^2 \rangle - \langle \chi \rangle^2 = A_{0,0}^{-1}/2 = \begin{cases} 1/4\gamma; & \gamma \gtrsim \gamma_c, \\ 4 - \frac{1}{2\gamma}; & \frac{2}{3}\gamma_c \lesssim \gamma \lesssim \gamma_c, \end{cases} \leq \frac{4}{3}. \quad (46)$$

The average value of the character $\chi^{(j)}$ in an arbitrary irreducible representation j may be computed as³

$$\langle \chi^{(j)} \rangle = \sum_{m=0}^{[j]} (-1)^m \binom{2j-m}{m} \left(\sqrt{-D/2} \right)^{2j-2m} H_{2j-2m} \left(\langle \chi \rangle / \sqrt{-2D} \right), \quad (47)$$

³ Here $[j]$ refers to an integer part of j and $H_n(x)$ are Hermitian polynomials of order n .

or

$$\langle \chi^{(j)} \rangle = \sum_{m=0}^{[j]} (-1)^m \binom{2j-m}{m} \langle \chi^{2j-2m} \rangle, \quad (48)$$

that for $\gamma \lesssim 2\gamma_c$, meaning $\langle \chi \rangle^2 \lesssim 2D$ immediately gives

$$\langle \chi^{(j)} \rangle \simeq \begin{cases} \frac{3}{4} \frac{(2j)!}{j!} \left(2 \left(1 - \frac{1}{8\gamma}\right)\right)^j \left(1 + j \frac{\langle \chi \rangle^2}{4 \left(1 - \frac{1}{8\gamma}\right)}\right); & [j] = j, \\ \langle \chi \rangle \frac{(2j)!}{(j-\frac{1}{2})!} \left(2 \left(1 - \frac{1}{8\gamma}\right)\right)^{j-\frac{1}{2}}; & [j] = j - \frac{1}{2}. \end{cases} \quad (49)$$

In the 'deep' deconfinement area $\gamma \gtrsim 2\gamma_c$ or $(\langle \chi \rangle^2 \gtrsim 2D)$

$$\langle \chi^{(j)} \rangle \simeq U_{2j} \left(\frac{\langle \chi \rangle}{2} \right) \simeq \langle \chi \rangle^{2j} \left(1 - \frac{j(j+1)}{\langle \chi \rangle^2} \right). \quad (50)$$

The pair correlation

$$\begin{aligned} \langle \chi_1^{(j_1)} \chi_2^{(j_2)} \rangle &= \overline{U_{2j_1} \left(\frac{\sigma_1}{2} \right) U_{2j_2} \left(\frac{\sigma_2}{2} \right)} = \frac{\sqrt{\det \alpha}}{\pi} \int_{-\infty}^{\infty} U_{2j_1} \left(\frac{\sigma_1}{2} \right) U_{2j_2} \left(\frac{\sigma_2}{2} \right) \\ &\quad \exp \{ -(\sigma_k - \langle \chi \rangle) \alpha_{k,r} (\sigma_r - \langle \chi \rangle) \} d^2 \sigma, \end{aligned} \quad (51)$$

where matrix $\alpha_{k,r}$ is given by

$$\begin{pmatrix} \alpha_{1,1} & \alpha_{1,2} \\ \alpha_{2,1} & \alpha_{2,2} \end{pmatrix} = \frac{1}{(A_{0,0}^{-1})^2 - (A_{0,R}^{-1})^2} \begin{pmatrix} A_{0,0}^{-1} & -A_{0,R}^{-1} \\ -A_{0,R}^{-1} & A_{0,0}^{-1} \end{pmatrix}, \quad (52)$$

one may do computation expanding over powers $A_{0,R}^{-1}$ and taking into account simple relation

$\chi^{(j)} \chi = \chi^{(j+\frac{1}{2})} + \chi^{(j-\frac{1}{2})}$. In particular for magnetization $M_{j,j'}$ we get

$$M_{j,j'} \equiv \langle \chi_0^{(j)} \chi_\infty^{(j')} \rangle = \langle \chi^{(j)} \rangle \langle \chi^{(j')} \rangle. \quad (53)$$

In the confinement area ($\gamma < \gamma_c$) for integer $j_{1,2} = [j_{1,2}]$ for $\alpha R \sim 1$

$$\langle \chi_1^{(j_1)} \chi_2^{(j_2)} \rangle \simeq \langle \chi^{(j_1)} \rangle \langle \chi^{(j_2)} \rangle \exp \left\{ \left(j_1 j_2 - \frac{1}{4} \right) \frac{2 (A_{0,R}^{-1})^2}{\left(8 - \frac{1}{\gamma} \right)^2} \right\}. \quad (54)$$

In particular for $j_1 = j_2 = j$ we obtain for $\alpha R \sim 1$

$$\left\langle \chi_1^{(j)} \chi_2^{(j)} \right\rangle \simeq \left\langle \chi^{(j)} \right\rangle^2 \exp \left\{ \frac{2 \left(A_{0,R}^{-1} \right)^2}{\left(8 - \frac{1}{\gamma} \right)^2} C_2 \right\} \sim \exp \left\{ - \frac{\alpha R C_2}{\left(4 - \frac{1}{2\gamma} \right)^2} \right\} \quad (55)$$

where $C_2 = \left(j + \frac{1}{2} \right)^2 - \frac{1}{4}$ is quadratic Casimir operator.

For half-integer $j_{1,2} = [j_{1,2}] + \frac{1}{2}$

$$\left\langle \chi_{x_1}^{(j_1)} \chi_{x_2}^{(j_2)} \right\rangle \simeq \frac{2 \left\langle \chi \chi^{(j_1)} \right\rangle \left\langle \chi^{(j_2)} \chi \right\rangle}{\left(A_{0,0}^{-1} \right)^2} A_{0,R}^{-1} = q(j_1; j_2) A_{0,R}^{-1}, \quad (56)$$

with

$$q(j_1; j_2) = \frac{2 \left(\left\langle \chi^{(j_1 + \frac{1}{2})} \right\rangle + \left\langle \chi^{(j_1 - \frac{1}{2})} \right\rangle \right) \left(\left\langle \chi^{(j_2 + \frac{1}{2})} \right\rangle + \left\langle \chi^{(j_2 - \frac{1}{2})} \right\rangle \right)}{\left(8 - \frac{1}{\gamma} \right)^2}, \quad (57)$$

and

$$\left\langle \chi^{(j - \frac{1}{2})} \right\rangle \simeq \frac{3}{4} \frac{(2j - 1)!}{\left(j - \frac{1}{2} \right)!} \left(2 \left(1 - \frac{1}{8\gamma} \right) \right)^{j - \frac{1}{2}}. \quad (58)$$

Therefore taking into account

$$\left\langle \chi^{(j)} \chi \right\rangle = \left\langle \chi^{(j + \frac{1}{2})} \right\rangle + \left\langle \chi^{(j - \frac{1}{2})} \right\rangle \simeq \langle \chi \rangle^{2j+1} + \langle \chi \rangle^{2j-1}, \quad (59)$$

we obtain

$$\begin{aligned} \left\langle \chi_{x_1}^{(j_1)} \chi_{x_2}^{(j_2)} \right\rangle &\simeq \left\langle \chi^{(j_1)} \right\rangle \left\langle \chi^{(j_2)} \right\rangle + \frac{2 \left\langle \chi \chi^{(j_1)} \right\rangle \left\langle \chi^{(j_2)} \chi \right\rangle}{\left(A_{0,0}^{-1} \right)^2} A_{0,R}^{-1} \\ &\simeq \langle \chi \rangle^{2j_1+2j_2} \exp \left\{ 8\gamma^2 \left(\langle \chi \rangle + \frac{1}{\langle \chi \rangle} \right)^2 A_{0,R}^{-1} \right\}. \end{aligned} \quad (60)$$

In particular we get

$$\left\langle \chi^{(1)} \right\rangle + 1 = \left\langle \chi^2 \right\rangle = \left[\left(\frac{\eta_x A_{x,0}^{-1}}{2} \right)^2 \right]_{\eta_x = \eta_0 \rightarrow 0} + \frac{1}{2} A_{0,0}^{-1} = \langle \chi \rangle^2 + \frac{1}{2} A_{0,0}^{-1}, \quad (61)$$

and

$$\begin{aligned}
\langle \chi_0^2 \chi_R^2 \rangle &= \left[e^{-\frac{1}{4}\eta A^{-1}\eta} \frac{\partial^2}{\partial \eta_R^2} \frac{\partial^2}{\partial \eta_0^2} e^{\frac{1}{4}\eta A^{-1}\eta} \right]_{\eta_x = \eta_0 \rightarrow 0} \\
&= \left(\langle \chi \rangle^2 + \frac{1}{2} A_{0,0}^{-1} \right)^2 + \frac{1}{2} \left(A_{R,0}^{-1} \right)^2 + \frac{3}{2} A_{0,R}^{-1} \langle \chi \rangle^2,
\end{aligned} \tag{62}$$

therefore, correlation function for characters in the adjoint representation $\langle \chi_0^{(1)} \chi_R^{(1)} \rangle$ may be written as

$$\langle \chi_0^{(1)} \chi_R^{(1)} \rangle \equiv \langle \chi_0^2 \chi_R^2 \rangle - 2 \langle \chi^2 \rangle + 1 = \langle \chi^{(1)} \rangle^2 + \frac{1}{2} \left(A_{R,0}^{-1} \right)^2 + \frac{3}{2} A_{0,R}^{-1} \langle \chi \rangle^2. \tag{63}$$

References

- M. Creutz, Phys. Rev. **D21** (1980) 2308.
- L. D. McLerran and B. Svetitsky, Phys. Lett. **98 B** (1981) 195.
- J. Kuti, J. Polónyi and K. Szlachányi, Phys. Lett. **98B** (1981) 199.
- G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, M. Lutgemeier, B. Petersson, Nucl.Phys. **B469** (1996) 419, BI-TP-96-04, hep-lat/9602007.
- J. Engels et al., Phys. Lett. **101B** (1981) 89.
- H. Satz, 'QCD & QGP: Summary', Theory Summary, *International Conference on the Physics and Astrophysics of the Quark-Gluon Plasma (ICPA-QGP'97)*, Jaipur/India, March 15 - 21.
- G. Mack, Phys. Lett. **B78** (1978) 263.
- A. M. Polyakov, Phys. Lett. **B72** (1978) 477;
- L. Susskind, Phys. Rev. **D20** (1979) 2610.
- P.H. Damgaard and M. Hasenbusch, Phys. Lett. **B331** (1994) 400
- L. Del Debbio, M. Faber, J. Greensite, and Š. Olejník, Phys. Rev. **D53** (1996) 5891, L. Del Debbio, M. Faber, J. Greensite, and Š. Olejník, Nucl. Phys. B Proc. Suppl. **53** (1997) 141.
- L. Del Debbio, M. Faber, J. Greensite, S.Olejnik, '*CENTER DOMINANCE, CASIMIR SCALING, AND CONFINEMENT IN LATTICE GAUGE THEORY*', hep-lat/9802003, L. Del Debbio, M. Faber, J. Greensite and Š. Olejník, '*CENTER DOMINANCE, CENTER VORTICES, AND CONFINEMENT*', hep-lat/9708023,. L. Del Debbio, M. Faber, J. Greensite and Š. Olejník, Nucl.Phys.Proc.Suppl.**63**(1998)552, hep-lat/9709032,
- H.D. Trottier, R.M. Woloshyn Phys.Rev.D48:2290-2298,1993,hep-lat/9303008, '*FLUX TUBES IN THREE-DIMENSIONAL LATTICE GAUGE THEORIES*'.
- J. Greensite and M.B. Halpern, Phys. Rev. **D27** (1983) 2545.
- C. Bernard, Nucl. Phys. **B219** (1983) 341, J. Ambjørn, P. Olesen and C. Peterson, **B240**[FS12] (1984) 189, 533.
- P. H. Damgaard, Phys. Lett. **B183** (1987) 81.
- M. Faber, H. Markum and M. Meinhardt, Phys. Rev. **D36** (1987) 632.
- M. Faber, J. Greensite, and Š. Olejník, '*Evidence for a Center Vortex Origin of the Adjoint String Tension*', hep-lat/9907008, M. Faber, J. Greensite, and Š. Olejník, '*Asymptotic Scaling, Casimir Scaling, and Center Vortices*', hep-lat/9809053'.
- G.I. Poulis and H.D. Trottier, archive: hep-lat/9504015; G.I. Poulis and H.D. Trottier, Nucl. Phys. **B** (Proc. Suppl.), **42**(1995) 267; G.I. Poulis, hep-lat/9601013.
- T.H. Berlin and M. Kac, Phys. Rev. **86** (1952) 82.
- L. Frachebourg and M. Henkel, hep-lat/9212012, '*Exact Correlation Function at the Lifshitz Points of the Spherical Model*'
- A.Cappelli and F.Colomo, hep-lat/971007.

- H. E. Stanley, Phys. Rev. **176** (1968) 718.
- M.N. Barber and M.E. Fisher, Ann. of Phys. **77** (1973) 1.
- S. Singh and R.K. Pathria, Phys. Rev. **B31** (1985) 4483, S. Singh and R.K. Pathria, **B36** (1987) 3769
- M. Zanetti and G.F. Mazenko, Phys. Rev. **B35** (1987) 5043; A. Coniglio and M. Zanetti, Europhys. Lett. **10** (1989) 575; T.J. Newman and A.J. Bray, J. Phys. **A23** (1990) 4491.
- A.M. Khorunzhy, B.A. Khoruzhenko, L.A. Pastur and M.V. Shcherbina in C. Domb and J. Lebowitz (Eds), *Phase Transitions and Critical Phenomena*, Vol. 15, Academic Press (New York 1992), p. 73 Les Houches XLIX, North Holland (Amsterdam 1990), p. 169.
- W. Selke, in C. Domb and J. Lebowitz (Eds), *Phase Transitions and Critical Phenomena*, Vol. 15, Academic Press (New York 1992), p.1.
- G.S. Joyce, Phase transitions and critical phenomena. V. 2. London, New York: Academic Press, 1972, Baxter, *Exactly Solvable Models in Statistical Mechanics*, Academic (1982), New York.
- F. Karsch, Nucl. Phys. **B 205**[FS5] (1982) 285.
- G. Burgers, F. Karsch, A. Nakamura, I.O. Stamatescu, Nucl. Phys. **B304** (1988) 587.
- B. Svetitsky and L. G. Yaffe, Nucl. Phys. **B210** [FS6] (1982) 423.
- M. Billo, M. Casselle, A. D'Adda, and S. Panzeri, DFTT 69/99, Nordita 96/1p (1996), 'Toward an analytic determination of the deconfinement temperature in $SU(2)$ L.G.T.' hep-lat 9601020.L.
- L. A. Averchenkova, K. V. Petrov, V. K. Petrov, G. M. Zinovjev, 'Asymmetry parameter role in description of phase structure of lattice gluodynamics at finite temperature, Yad. Phys. **60** (1997)1, L. A. Averchenkova, K. V. Petrov, V. K. Petrov, G. M. Zinovjev, 'Lattice asymmetry in finite temperature gluodynamics, Phys. Rev. **D56**, v.11(1997)56.
- A. Hasenfratz and P. Hasenfratz. Nucl. Phys. **B193**, 210 (1981).
- J. Shigemitsu, J. B. Kogut. Nucl. Phys. **B190**, 3650 (1981).
- F. Karsch. 'SU(N) GAUGE THEORY COUPLINGS ON ASYMMETRIC LATTICES', Nucl.Phys.B205:285-300,1982.
- G. Burgers, F. Karsch, A. Nakamura, and I.O. Stamatescu, Nucl. Phys. **B304** (1988) 587.
- J. Engels, F. Karsch and T. Scheideler, Nucl. Phys. **B** (Proc. Suppl.) **63** (1998) 427.
- T.R. Klassen, hep-lat/9803010.
- J. Engels, F. Karsch and K. Redlich, Nucl. Phys. **B435** (1995) 295.
- G. Boyd et al., Nucl. Phys. **B469** (1996) 419.
- S. Ejiri, Y. Iwasak and K. Kanaya, 'Non-perturbative determination of anisotropy coefficients and pressure gap at the deconfining transition of QCD' 9808034.
- F. Green and F. Karsch, Nucl. Phys. **B238 34**, (1984) 297.

- M. Ogilvie, Phys. Rev. Lett. **52** (1984)1369, '*AN EFFECTIVE SPIN MODEL FOR FINITE TEMPERATURE QCD*'.
- A. Ukawa, UTHEP-302, Uehling Summer School (1993):0211-304, '*LECTURES ON LATTICE QCD AT FINITE TEMPERATURE*'.
- G. Mack, and V.B. Petkova, Ann. Phys. **123** (1979) 442; *ibid* **125** (1980) 117; Z. Phys. **C12** (1982) 177.
- L.G. Yaffe, Phys. Rev. **D21** (1979) 1574, *ibid* **D21**, 1574, (1980).
- E.T. Tomboulis, Phys. Rev. **D23**, 2371, (1981), E.T. Tomboulis, Phys. Lett. **B303**, 103, (1993); Nucl. Phys. **B34** (1994) 192, (Proc. Suppl.); T.G. Kovács and E.T. Tomboulis, Nucl. Phys. **B53** (1997) 509, (Proc. Suppl.).
- E.L. Gubankova and Yu.A. Simonov, hep-lat 9508206.
- U.M. Heller, F. Karsch, J. Rank, Phys.Lett.**B355** (1995) 511.
- J. Engels, V. K. Mitrjushkin and T. Neuhaus, hep-lat 9311055 and hep-lat 9412003.
- G. Cella, V.K. Mitrjushkin and A. Viceré, '*On T dependence of the static potential $V(T; \vec{R})$ in a finite volume*' hep-lat 9412017.
- G. Cella, U.M. Heller, V.K. Mitrjushkin and A. Viceré, hep-lat 9704012.
- P. Damgaard, Phys. Lett. **B194** (1987) 107.
- K. Redlich and H. Satz, Phys. Rev. Lett. **B213** (1988)191.
- J. Kiskis (UC, Davis). 1990. In *Houston 1990, Proceedings, Rice Meeting, vol. 2* 793.
- K. Binder and D. P. Landau, Phys. Rev. **B30** (1984) 1477.